Exam Paper (bachelor/specialist) No. 11
Subject: Mathematics

Instructions

You have 4 hours to complete the exam. The exam paper consists of 2 parts and contains 15 tasks.

Do not copy the text of the tasks in your answer sheet. All tasks must be solved in a draft answer sheet. Tasks, which are not solved in a draft answer sheet, are not accepted.

For each task solved in the answer sheet write the full reasoned solution and the answer.
The answers in the answer sheet for the Part 1 must be written in the Answer Table shown below.

Answer Table for Part I

Task No.	1	2	3	4	5	6	7	8	9	10	11	Total score
Answer												
Points												

Attention! In your answer sheet fill in only the row with answers, the points and total score should be left blank. Below or under the table write a full solution and the answer to each task.

Part I

1. There is a tradition to present an odd number of flowers to a person's birthday. Chrysanthemums cost 65 rubles per flower. What is the maximum number of flowers that Artem can buy for his girlfriend's birthday if he has 800 rubles?

   (5 points)

2. Anna pays 4500 rubles per month for utilities. Next year the cost will rise by 18%. How much will Anna have to pay each month next year?

   (5 points)

3. Evaluate the expression $\qquad 5 - \left(\frac{3\sqrt{5}}{2} - 1\right) \cdot \left(\frac{3\sqrt{5}}{2} + 1\right).$

   (5 points)

4. Simplify the expression: $\qquad \left(\frac{2}{x} - \frac{x-2}{x^2-x}\right) : \left(\frac{3}{x} + \frac{x+3}{x^2-x}\right).$

   (6 points)

5. Solve the equation: $\qquad \left(\frac{5}{6}\right)^{1-5x} = \left(\frac{6}{5}\right)^{2+x}.$

   (6 points)

6. Find the number of integers in the domain of the function:
   $$f(x) = \frac{3}{\sqrt{15 + 3x}} - 5\sqrt{2 - 9x}.$$

   (6 points)

7. Find the sum of all integer solutions of the inequality: $\qquad \frac{4-x}{x-5} > \frac{1}{1-x}.$

   (6 points)

8. The perimeter of a rhombus equals to 120 and one of the angles is 30°. Find the area of the rhombus.

(6 points)

9. Find the surface area of the right triangular prism the height of which is 6, at its base is a regular triangle with the hypotenuse that equals 10 and one of the catheti (legs) equals 8.

(6 points)

10. First pipe passes 4 liters of water per minute less then second pipe. How many liters of water per minute passes through the first pipe if it fills a 480 liter basin 8 minutes longer than the second pipe fills a 384 basin?

(7 points)

11. Find the point of minimum of the function: $y = -x^3 - 3x^2 + 10$.

(7 points)

Part II

12. Solve the equation $sin2x = \sqrt{3}cos\left(\frac{3\pi}{2} - x\right)$ and write the roots of this equation that belong to the interval $\left[\pi; \frac{7\pi}{2}\right]$.

(8 points)

13. Solve the inequality: $log_4(3 \cdot 4^{x+1} - 8) \leq 2x + 1$.

(8 points)

14. The sides of the pyramid *MNKL* are acute triangles, where *NK*=17, *LK*=14, $\angle MLK = 60°$. Find the length of the edge *LN*, if all the altitudes are equal.

(9 points)

15. Find all the values of the argument a with which the equation
$$tg\left(2\pi\left(x + \frac{1}{2}\right)\right) \cdot lg(4x + a + 3) = lg(4x + a + 3)$$
has the only root in the interval $\left[-\frac{1}{2}; 0\right]$.

(10 points)

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